Dynamic coordinated control laws in multiple agent models

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Abstract

We present an active control scheme of a kinetic model of swarming. It has been shown previously that the global control scheme for the model, presented in [1], gives rise to spontaneous collective organization of agents into a unified coherent swarm, via a long-range attractive and short-range repulsive potential. We extend these results by presenting control laws whereby a single swarm is broken into independently functioning subswarm clusters. The transition between one coordinated swarm and multiple clustered subswarms is managed simply with a homotopy parameter. Additionally, we present as an alternate formulation, a local control law for the same model, which implements dynamic barrier avoidance behavior, and in which swarm coherence emerges spontaneously.

Key words: swarming, control, dynamics, emergent behavior

1 Introduction

Multiple agent models are comprised of a multitude of simple autonomous vehicles, which are loosely coupled via communication. It is anticipated that such systems will play a key role in future deployments, as the drive to miniaturize electronic devices results in smaller and more capable self-mobile machines with limited decision making abilities. Thus, one of the main research areas of interest is the dynamic pattern formation and control of a large number of agents [2]. In particular, given a specific dynamical system composed of a large number of individual vehicles, each with specified limited decision-making and communication abilities, a vital question is under what conditions large-scale

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aggregate dynamics may be controlled to form coherent structures, or patterns. An example from electronics is a concept paper [3] which shows that complex patterns can arise from a large array of micro actuators interconnected to mimic a finite difference approximation of standard reaction diffusion partial differential equations (PDE). However, it is a static theory based on quite standard pattern formation theories from reaction-diffusion which assumes pure local coupling.

In contrast, many biological examples of coherent dynamical motion (swarming) exist in nature. Populations such as bees, locusts, and wolves often move in coordinated but localized efforts toward a particular target. In addition many more examples abound of populations of individuals that move according to local rules, and whose aggregate dynamics achieve an overall large-scale complex pattern or state. Bacterial colonies, which evolve in part via chemotactic response, are such an example. The mathematical biology community has been exploring models for animal swarms, and this work pinpoints some of the difficulties (see the survey paper [4]). Traditional models for biology populations involve local PDE for the population density [5]. Edelstein-Keshet *et al* [6] recently considered such a model in one space dimension for African migratory locusts. These insects have a gregarious phase in which swarms of individuals can travel for days over thousands of miles. Evidence exists that the swarms remain cohesive even in the absence of a nutrient gradient. The analysis of [6] shows that such cohesive swarms cannot be described by traveling wave solutions of their one dimensional advection-diffusion model. More recently, Mogilner and Edelstein-Keshet consider nonlocal interactions, in which the drift velocity of the population is determined by a convolution operator with the entire population [7]. These models, resulting in integro-differential equations, do sometimes produce coherent band-like structure. Earlier work by Edelstein-Keshet and Watmough [8] on army ant swarms, considers a one dimensional model and shows the existence of traveling wave solutions for the leading edge of the pack, but they do not consider band-like solutions that would describe something like a locust swarm. These particular examples involve one-dimensional models and simulations. In summary, most studies of biological swarming involve models from continuum theory, many of which are based on some form of local communication, which are modeled by way of interactions or couplings.

The statistical physics community has recently tried to understand similar problems in situations where the number of individuals are very large. Statistical information derived for large numbers is less relevant to sensor applications involving smaller numbers of individuals. However, the connection between the discrete and the continuous is an important problem that is well-studied in this field. The particle approach involves starting with simple rules of motion, involving combinations of biased random walks, sampling of motions and positions of nearby neighbors, with some governing strategy designed to mimic

core components of animal interactions. For example, Schweitzer et al [9] consider a theory of canonical-dissipative systems and the energetic conditions for swarming. Daniel Grünbaum [10] has derived advection diffusion equations for internal state-mediated biased random walks. Mogilner and Edelstein-Keshet [11] consider both continuum and cellular automata models for populations of self-aligning objects. Stöcker [12] considers a hexagonally based cellular automata model for tuna school formation. These are just a few examples. In all cases, the local rules are precisely defined and aggregate motion can be observed in numerical simulations.

As an alternative to understanding coherent swarm structures that use finite models (non-continuum theories), a recent body of work considers general particle-based models for self-propelled organisms (see for example [13,14,15,16]). Collective motion and swarming is observed along with interesting aspects of dynamic phase transitions, including crystalline like motion, liquid, solid, and gas-like states. Toner and Tu [17,18,19] use renormalization group ideas to study flocking motion in a particle-based model. Some of this work parallels classical statistical theory of transport which derives hydrodynamic equations from local interaction models [20,21,22]. The approach considered by Chang, *et al.* [23] considers agents in a scalar potential field and utilizes gyroscopic and braking forces.

In most cases presented, the agents are self-propelled and the nature of the coupling or communication imposes a given pattern. Here we consider similar aspects, but with the idea of controlling the communication to form patterns. In this article we consider kinetic models in which, depending on the control law used, the self-propelled agents communicate, either locally within a specified radius about each agent, or globally with every other agent in the swarm. Under appropriate choices of controlling "potentials", coherent motion of agents is observed. In general, the models considered are based on controls which involve long range attraction and short range repulsion, similar to the ideas in [24]. However, in [24], the computational approach to obstacle avoidance, achieved by forming clustered groups from a single coherent swarm, is to use genetic algorithms, which contain a number of restrictive rules. This violates the assumption of creating a swarm with limited computational ability.

In the work presented here, we consider the problem of dynamically deforming a single large and coherent swarm into a collection of subswarm clusters under simple control modifications. A cluster is a subset of the original swarm which functions independently as a coherent swarm, and which, when fully formed, does not interact with agents that are not members of the cluster. A primary goal of this work is to generate simple algorithmic controls for obstacle avoidance, and we consider two methods to achieve this. We also consider multiple approaches to guiding a swarm, by dynamically steering leader agent(s), and by *a priori* fixing a target to which all agents are attracted.

We formulate the first control problem using homotopy, or continuation, theory [25,26]. The homotopy parameter controls the communication coupling, selecting between local and global communication, and may simultaneously be used to modify other characteristics of the control law. Such a control law allows one to use a single parameter to switch from a single coherent swarm state (global coupling) to a multiple cluster state (local coupling) and back again. Swarm coherence and inter-agent collision avoidance is achieved with this control law via a long-range attractive/short-range repulsive potential, and swarm navigation is implemented via group-averaged motion and leader-following controls.

We also consider an alternate formulation using only local coupling between agents, in which a convex barrier is detected and avoided, and where the barrier location is not *a priori* known. This approach to obstacle avoidance is similar in nature to that discussed in [23]. In addition, swarm navigation is achieved by introducing terms in the control law so that all agents seek a common target. Whereas clustering with the homotopy control law is due to an attractive potential to other agents, clustering appears to arise naturally with this control law, as agents interact while they seek out a common target.

The layout of the paper is as follows: In section 2 we introduce the kinetic model presented in [1], and discuss some important properties of its global control law. In Section 3 we introduce a modified control law implementing local coupling. A homotopy control law is presented in Section 4. In Section 5 we present the alternative control law with an alternate approach to target seeking behavior and barrier avoidance, and we conclude with a discussion.

2 Multi-agent kinetic model and properties

The ideas we present apply to a large class of systems. Consider a continuous dynamical system $\frac{d\mathbf{z}}{dt} = \mathbf{F}(\mathbf{z}(t))$ arising from an autonomous vector field $\mathbf{F}$, where $t \in \mathbb{R}$ and $\mathbf{z} \in \mathbb{R}^n$, describing the equations of motion. Associated to this dynamical system is the system governing trajectories, in which all orbits have unit velocities,

$$\frac{d\mathbf{r}}{dt} = \mathbf{G}(\mathbf{r}(t)) \equiv \frac{\mathbf{F}(\mathbf{r})}{\|\mathbf{F}(\mathbf{r})\|}. \tag{1}$$

Consider a (nontrivial) trajectory $\mathbf{r}(t)$ of (1), its associated unit tangent vector $\mathbf{x} = \mathbf{G}(\mathbf{r}(t))$ defined for all $t \in \mathbb{R}$, and the positively oriented unit conormal vectors $\mathbf{y}_i(t)$, $(i = 1, n-1)$. The collection of vectors $\mathcal{F} = \{\mathbf{x}(t), \mathbf{y}_i(t)\ (i = 1, \ldots, n-1)\}$ is called the (moving) reference frame associated to $\mathbf{r}$. Thus, one

may recast a continuous dynamical system as a system of trajectories $\mathbf{r}(t)$ parameterized by arclength, with the associated moving frame $\mathcal{F}$. The behavior of a system of trajectories with the associated moving frame is governed by the Frenet-Serret system of equations.

2.1 Derivation of Frenet-Serret system of equations

We derive the Frenet-Serret equations, restricted to the plane, following the approach of [27]. Consider a differentiable trajectory $\mathbf{r}(t)$ in $\mathbb{R}^2$, parameterized by arclength, which represents the motion of an agent over time. A positively oriented orthonormal frame $\mathbf{x}$ and $\mathbf{y}$ is associated to $\mathbf{r}(t)$, by taking $\mathbf{x}$ equal to the unit tangent vector $d\mathbf{r}/dt$, and $\mathbf{y} = \mathbf{x}^{\perp}$ to be the unit normal vector positively oriented relative to $\mathbf{x}$. There exists a function $\kappa(t)$, called the curvature of $\mathbf{r}(t)$, such that

$$\frac{d\mathbf{r}}{dt} = \mathbf{x}(t), \quad \frac{d\mathbf{x}}{dt} = \kappa(t) \cdot \mathbf{y}(t). \tag{2}$$

One then obtains the equation governing the unit normal vector $\mathbf{y}$ as follows. Using the right-hand equation of Eq. (2) we find that $\mathbf{x} \cdot \mathbf{y} = d\mathbf{x}/dt \cdot \mathbf{y} + \mathbf{x} \cdot d\mathbf{y}/dt = \kappa(t) + \mathbf{x} \cdot d\mathbf{y}/dt = 0$, and thus $d\mathbf{y}/dt = -\kappa(t) \cdot \mathbf{x}$. The moving frame $\mathbf{x}$ and $\mathbf{y}$ associated with $\mathbf{r}(t)$ can be expressed by a rotational matrix $R(t)$, which has columns consisting of coordinates of $\mathbf{x}$ and $\mathbf{y}$ relative to a fixed orthonormal frame e_1 and e_2 in $\mathbb{R}^2$.

This formulation leads to a natural Lie group setting, but we do not consider that aspect further in this article. We also note that it is also possible to derive the Frenet-Serret equations by considering the problem of steering unit-charge, unit-mass particles in a magnetic field. For details, consult [1] and references therein.

2.2 Equations of motion for multiple agents.

We consider a set of n agents, restricted to smooth motions in the plane, and moving at unit speed. The system of equations modeling each agent is

$$\begin{aligned} \dot{\mathbf{r}}_k &= \mathbf{x}_k \\ \dot{\mathbf{x}}_k &= \mathbf{y}_k \cdot u_k \\ \dot{\mathbf{y}}_k &= -\mathbf{x}_k \cdot u_k, \end{aligned} \tag{3}$$

for $k=1,\ldots,n$. The orientation of an agent is given by the moving frame $\mathbf{x}$ and $\mathbf{y}$, its trajectory is given by $\{\mathbf{r}(t)|t\in\mathbb{R}\}$, and the agents are coupled together via a scalar curvature control law u, which is detailed below.

The control law u_k, introduced in [1], is

$$u_k = \sum_{j\neq k} u_{jk}, \tag{4}$$

with

$$u_{jk} = \left[-\eta\left(\frac{\mathbf{r}_{jk}}{|\mathbf{r}_{jk}|}\cdot\mathbf{x}_k\right)\left(\frac{\mathbf{r}_{jk}}{|\mathbf{r}_{jk}|}\cdot\mathbf{y}_k\right) - f\left(|\mathbf{r}_{jk}|\right)\left(\frac{\mathbf{r}_{jk}}{|\mathbf{r}_{jk}|}\cdot\mathbf{y}_k\right) + \mu\mathbf{x}_j\cdot\mathbf{y}_k\right] \tag{5}$$

where $\mathbf{r}_{jk}\equiv\mathbf{r}_k-\mathbf{r}_j,\ f$ is

$$f\left(|\mathbf{r}_{jk}|\right) = \alpha\left[1-\left(\frac{r_0}{|\mathbf{r}_{jk}|}\right)^2\right], \tag{6}$$

and $\eta=\eta(|\mathbf{r}|),\ \mu=\mu(|\mathbf{r}|)$, and $\alpha=\alpha(|\mathbf{r}|)$ are specified functions. We now describe this control law in some detail. We first note that when $u_k<0$ ($u_k>0$), the Frenet frame will rotate in a clockwise (anticlockwise) fashion, respectively. In order to simplify the following discussion, we consider the case of $n=2$, but note that the discussion holds for general n. Let $\mathbf{r}_1$, $\mathbf{x}_1$ and $\mathbf{y}_1$ be the position and corresponding Frenet frame of one of the agents. We will examine each of the terms in Eq. (5) in turn. The first term, $-\eta(\mathbf{r}_{jk}/|\mathbf{r}_{jk}|\cdot\mathbf{x}_k)(\mathbf{r}_{jk}/|\mathbf{r}_{jk}|\cdot\mathbf{y}_k)$ serves to orient the vehicles perpendicular to their common baseline, $\mathbf{r}_{jk}$. To see this, let $\theta_{\mathbf{x}}$ and $\theta_{\mathbf{y}}$ be the angles the (unit) vectors $\mathbf{x}_1$ and $\mathbf{y}_1$ make with $\mathbf{r}_{21}/|\mathbf{r}_{21}|=(\mathbf{r}_1-\mathbf{r}_2)/|\mathbf{r}_1-\mathbf{r}_2|$, respectively. Then

$$\begin{aligned}-\eta(\mathbf{r}_{21}/|\mathbf{r}_{21}|\cdot\mathbf{x}_1)(\mathbf{r}_{21}/|\mathbf{r}_{21}|\cdot\mathbf{y}_1) &= -\eta\cos(\theta_{\mathbf{x}})\cos(\theta_{\mathbf{y}})\\ &= -\eta\cos(\theta_{\mathbf{x}})\cos(\theta_{\mathbf{x}}-\frac{\pi}{2}).\end{aligned}$$

This expression is zero for $\theta_{\mathbf{x}}=\frac{\pi}{2},\frac{3\pi}{2}$, positive for $0\leq\theta_{\mathbf{x}}<\frac{\pi}{2}$ and $\pi\leq\theta_{\mathbf{x}}<\frac{3\pi}{2}$, and is negative elsewhere. Thus, this term steers the vehicle to the nearest perpendicular with the baseline $\mathbf{r}_{21}$.

Inter-agent spacing is controlled via the short-range repulsive/long-range attractive term,

$$-\alpha\left[1-\left(\frac{r_0}{|\mathbf{r}_{jk}|}\right)^2\right]\left(\frac{\mathbf{r}_{jk}}{|\mathbf{r}_{jk}|}\cdot\mathbf{y}_k\right). \tag{7}$$

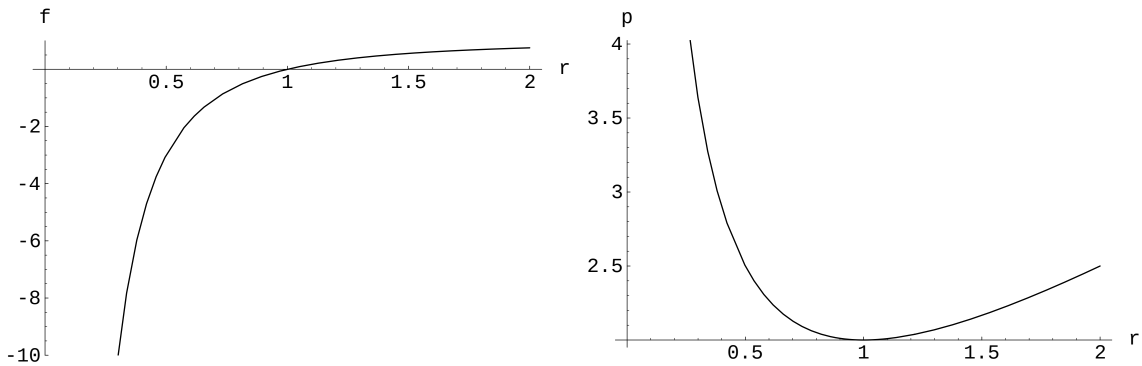

Figure 1. This shows the function $f(|r_{jk}|) = \alpha(1 - [r_0/|r_{jk}|]^2)$, and the corresponding (unbounded) potential p, for $r_0 = 1$ and $\alpha = 1$. Two agents greater in distance than r_0 are attracted to one another, while they are strongly repelled when the distance is less than r_0.

The first factor of (7), which arises from a Leonard-Jones type of potential, is negative if the distance between two agents is less than r_0, and positive if the distance is greater than r_0. The sign of the second factor is determined by the orientation of the two agents $\mathbf{r}_j$ and $\mathbf{r}_k$, relative to the baseline between them. See figure 1, which shows both the graphs of the potential, and of f.

It is easy to see that the third term, $\mu \mathbf{x}_j \cdot \mathbf{y}_k$, serves to drive the vehicles to a common orientation, by rewriting the dot product in terms of cosines.

The control law (4) is global (see Fig. 2), meaning that every agent communicates with all other agents in the swarm. Furthermore, the final orientation (heading) of the swarm is obtained by group averaged motion, which is in turn determined by the initial positions and orientations of the agents. We define this as the globally coupled, group averaged motion law. For the case $n = 2$, rigorous global convergence results have been obtained, by reducing (3) via the symmetry group SE(2) and demonstrating explicitly the existence of a Lyapunov function, the physical result being that agents will align to the same heading, perpendicular to their common base-line, and with the appropriate distance between them. See [1] (and [28]) for details. Recently, local convergence results were obtained for the general case of n agents. See [29] for details.

3 The leader following control law utilizing local coupling.

The control law (4) is global; that is, at each time-step, an agent requires information from all other agents in the swarm. Global communication is however often not practical. It is a goal to miniaturize mobile platforms as much as possible, and so not surprisingly, space constraints limit the power and sensitivity of on-board sensors and transmitters. Environmental factors, such as

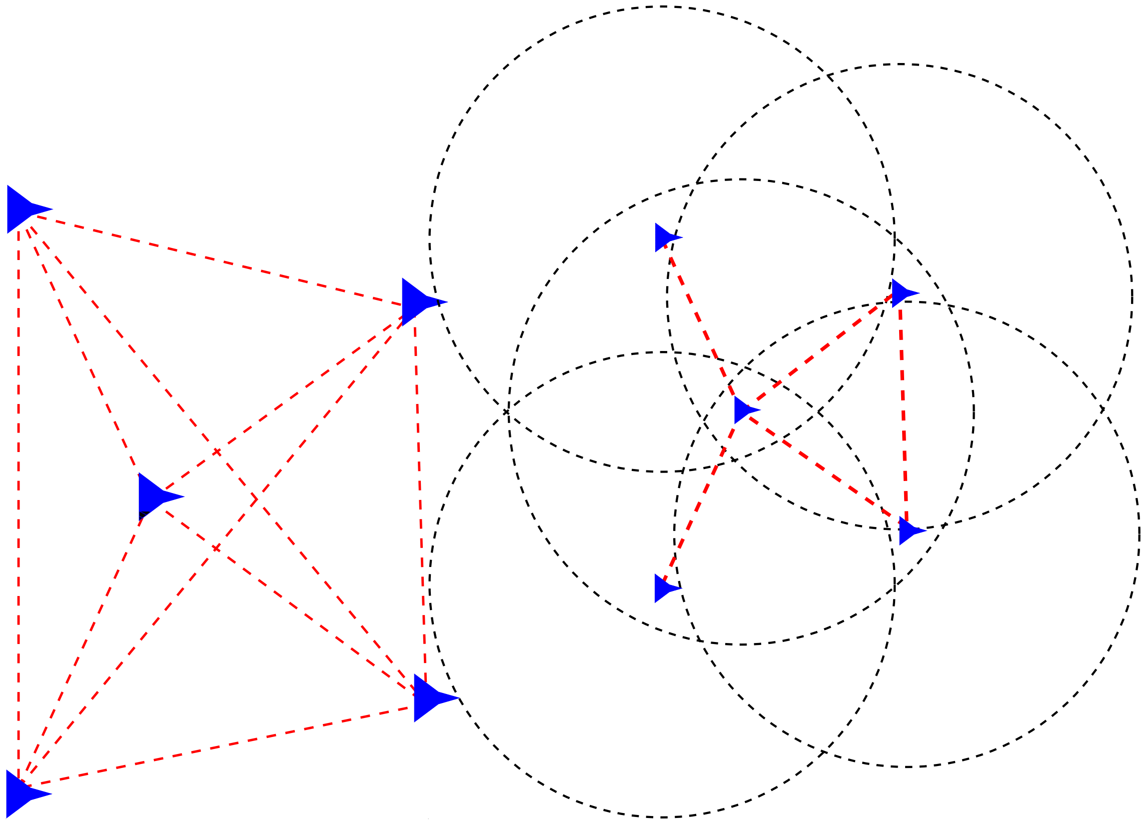

Figure 2. The left figure shows the global coupling implemented by the control law (4). The right figure shows an example of local coupling scenario implemented by the control law (9). Dashed lines indicate agents which are coupled to one another, while the circles indicate the maximal communication radius. Note that each agent is path connected to every other agent. For example, the upper left and lower right agents are not directly connected, but they are path connected, since they are both in range of the central agent.

weather effects and local geography can also have detrimental effects on electromagnetic signals. On the other hand, a local control law only requires an agent to communicate with some subset of agents in the swarm, such as their nearest neighbors. Indeed local coupling is observed in most natural swarms, such as schooling fish and flocking birds. See Fig. 2 for a comparison of the global and local coupling we employ.

We employ local coupling of agents by limiting communication to a neighborhood of each agent, so that there may be agents that are not in communication with other agents in the swarm. However, we choose initial conditions such that each agent is in communication range of at least one other agent, and such that all agents are 'path connected' initially, meaning that any two agents in the swarm are coupled at least through intermediary agents. See Fig. 2 and caption.

The implementation of the local coupling model is straight-forward. We simply

multiply the control law (4) with the cutoff function

$$c\left(\left|\mathbf{r}_{jk}\right|,q,w\right)=\begin{cases}1 \text{ if } \left|\mathbf{r}_{jk}\right|<w,\\ q \quad \text{otherwise.}\end{cases} \tag{8}$$

By using a nonzero value for q, one obtains a global 'cutoff' function, that is useful for imposing stronger local coupling, while maintaining weak global coupling (by setting $q \ll 1$). For the present discussion we set $q=0$, so that only when the Euclidean distance between two agents is less than w will they interact. We thus obtain the modified law

$$u_k^L=c\left(\left|\mathbf{r}_{jk}\right|,0,w\right)\sum_{j\neq k}u_{jk}, \tag{9}$$

where u_{jk} is defined by Eq. (5) .

We note that when the distance between all agents is less than w, Eq. (9) reduces to the original control law (4), while if the swarm is split into subswarm clusters greater than distance w from one another, the subswarms will evolve independently of one another.

The control law (4) uses group averaged motion for swarm control. The asymptotic heading of the swarm is thus determined by the initial conditions. We wish to control the direction of the swarm without having to steer each agent individually. We implement a leader following control which allows one to 'steer' the swarm by controlling a designated leader agent (or agents). This provides simple directional control of a swarm, since only the leader agents are steered, and the nonleader agents, which we define to be follower agents, pursue leader agents automatically. We note that leader following behavior can be implemented with either local or global agent coupling.

The leader following, local control law is obtained by using (9) for follower agents, modified so there is stronger coupling between follower and leader agents,

$$u_k^{follower}=c\left(\left|\mathbf{r}_{l(k)k}\right|,0,w\right)\ell_c u_{l(k)k}+\sum_{j\neq k,l(k)}c\left(\left|\mathbf{r}_{jk}\right|,0,w\right)u_{jk}, \tag{10}$$

where ℓ_c is a coupling constant and $l(k)$ is the index of the leader swarmer closest to the k^{th} follower swarmer, while for leader agents the control law is simply

$$u_k^{leader}=s_k, \tag{11}$$

where s_k is an explicit steering program, which can be given by a trajectory from a dynamical system.

4 Homotopy control law.

We combine the leader following, local control law given by Eqs. (10) and (11), introduced in the previous section, with the group averaged, global control law (4) of section 2 to obtain a hybrid control law utilizing a homotopy parameter (defined below), which we hereafter refer to as a homotopy control law. The introduction of a homotopy parameter provides a simple mechanism to dynamically switch from one control law to another.

Let u_k^G be the global control law (4) and let u_k^L be the local control law given by Eqs. (10) and (11). The homotopy control law

$$u_k = u_k(\lambda),\ 0 \le \lambda \le 1 \tag{12}$$

is defined by the properties

$$u_k(\lambda = 0) = u_k^G, \quad u_k(\lambda = 1) = u_k^L,$$

along with the property that the control law u_k varies smoothly with λ.

To implement the homotopy control law (12), we designate m agents to be leaders, so that there will be $n-m$ follower agents. Additionally, let $l(k)$ be the index associated to the closest leader (in the Euclidean sense) of the k^{th} follower agent. The homotopy control law for follower agents is

$$\begin{aligned} u_k^{follower}(\lambda) = {} & c\left(\left|\mathbf{r}_{l(k)k}\right|, 1-\lambda, w\right)[(\ell_c - 1)\lambda + 1] u_{l(k)k} + \\ & \sum_{j \neq k, l(k)} c\left(\left|\mathbf{r}_{jk}\right|, 1-\lambda, w\right) u_{jk} \end{aligned} \tag{13}$$

where u_{jk} is given by (5) and $\ell_c \gg 1$ is a constant which couples followers more strongly to their closest leader agent. This coupling constant was found to be necessary for the proper swarm-splitting behavior to emerge. The follower agents must react more strongly to the motion of leader agents, otherwise some follower agents were observed to escape from a local neighborhood of the swarm, and would thus no longer interact with it. The homotopy control law for the leader agent(s) is

$$u_k^{leader}(\lambda) = \sum_{j \neq k} \left[u_{jk}(1-\lambda) + \frac{s_k}{n-1}\lambda \right], \tag{14}$$

where s_k is an explicit steering program, which may be supplied by an external dynamical system.

When $\lambda = 0$, the cutoff function $c\left(\left|\mathbf{r}_{jk}\right|, 1, w\right) \equiv 1$, and both the follower control law (13) and leader control law (14) reduce to the global control law (4). When $\lambda = 1$ on the other hand,

$$c\left(\left|\mathbf{r}_{jk}\right|, 0, w\right) = \begin{cases} 1 \text{ if } \left|\mathbf{r}_{jk}\right| < w, \\ 0 \text{ otherwise,} \end{cases}$$

and inter-agent coupling is local. The control law for the follower and leader agents is in this case given by Eqs. (10) and (11), respectively.

When only one leader is present, the swarm transitions from group averaged motion to leader following motion, and thus the swarm can be directed, as $\lambda \to 1$. Note that in this case, the local coupling plays no significant role in the behavior, other than perhaps making the swarm more robust to communications difficulties.

When more than one leader agent is designated, other swarming behaviors are possible. In this case, the local coupling plays a crucial role. For example, assume there are two leader agents. As $\lambda \to 1$, and the two leaders are directed away from one another, the swarm will effectively split into two subswarm clusters. This is the result of the local coupling and the fact that follower agents are more strongly coupled to their *closest* leader agent. As the leader agents diverge, the follower agents following one leader leave the communications range of the followers of the other leader, so that they no longer interact. The sets of equations modeling the two subswarms in this case decouple.

4.1 Homotopy control law simulation results

We present the results of simulations of the modified model using the homotopy control law (12). We additionally present the results of simulations using the homotopy control law with local coupling throughout. That is, we consider the homotopy control law (12) with the modification

$$u_k(\lambda = 0) = u_k^{GL},$$

where u_k^{GL} utilizes group averaged motion, but with local coupling. For each control law, we ran several simulations, each with random initial data, as described below.

We first present a prototypical simulation using the homotopy control law presented in section 4, using $n = 6$ agents in the swarm. The parameters are $\alpha = \eta = \mu = 0.02$ and $r_0 = 1.5$, while the cutoff function parameters are $q = 0$ and $w = 4$, and simulations commence with $\lambda = 0$, so that the initial control law is the globally coupled group averaged law (4). The initial positions of the six agents are $\{(-1,1),(0,1),(1,1),(-1,0),(0,0),(1,0)\}$, while the initial orientations θ_i of each agent is randomly chosen from angles constrained to lie $\phi - \pi/4 < \theta_i < \phi + \pi/4$, and $\phi \in [0,2\pi]$ is also randomly chosen, and the system is integrated to $t = 1000$. Near $t = 400$, the homotopy parameter λ increases to one, and the control laws (10) and (11) are used. See figure 3. As the homotopy parameter is switched on, the two leaders use the simple 'programs' $s_1 = 0.01$ and $s_2 = -0.01$. This causes one leader to start a gentle clockwise loop and the other leader to begin a gentle counterclockwise loop. At this point in the simulation, the entire swarm then splits into two subswarms as the follower agents move toward their nearest leader agent. As the homotopy parameter is decreased to zero near $t = 600$, the control law reverts to (4), global communication between all agents is restored, and the two subswarms reform as one swarm.

We next present an extension of the above, a prototypical simulation using the homotopy control law, but with local coupling throughout, so that the simulation commences with locally coupled, group averaged motion, with all agents in communications range of one another. As in the previous subsection, the homotopy parameter is initially zero, but near $t = 400$ it is increased to one, and the system transitions from group averaged motion to leader following motion, with two leaders. Once again the swarm splits into two subswarm clusters for the same reasons as outlined in the previous subsection. Near $t = 600$, the homotopy parameter is then decreased to zero, and the system returns to group averaged motion. See figure 4. However, due to the local nature of the coupling, once the system returns to group averaged motion, the two subswarms remain independent, since the subswarms are not in communications range of one another.

5 Target seeking and barrier avoidance control laws

The homotopy control law of the preceding section provided a mechanism to switch between global and local coupling, and thus created a swarm which could split into subswarms, and each of the subswarms could be independently directed. We now introduce an additional method for both swarm navigation, by coding an *a priori* target-seeking behavior into the control law, and a method of barrier avoidance, by automatically sensing a barrier and splitting the swarm around the barrier. We note that control laws presented in this section may also be integrated with the previously introduced control laws.

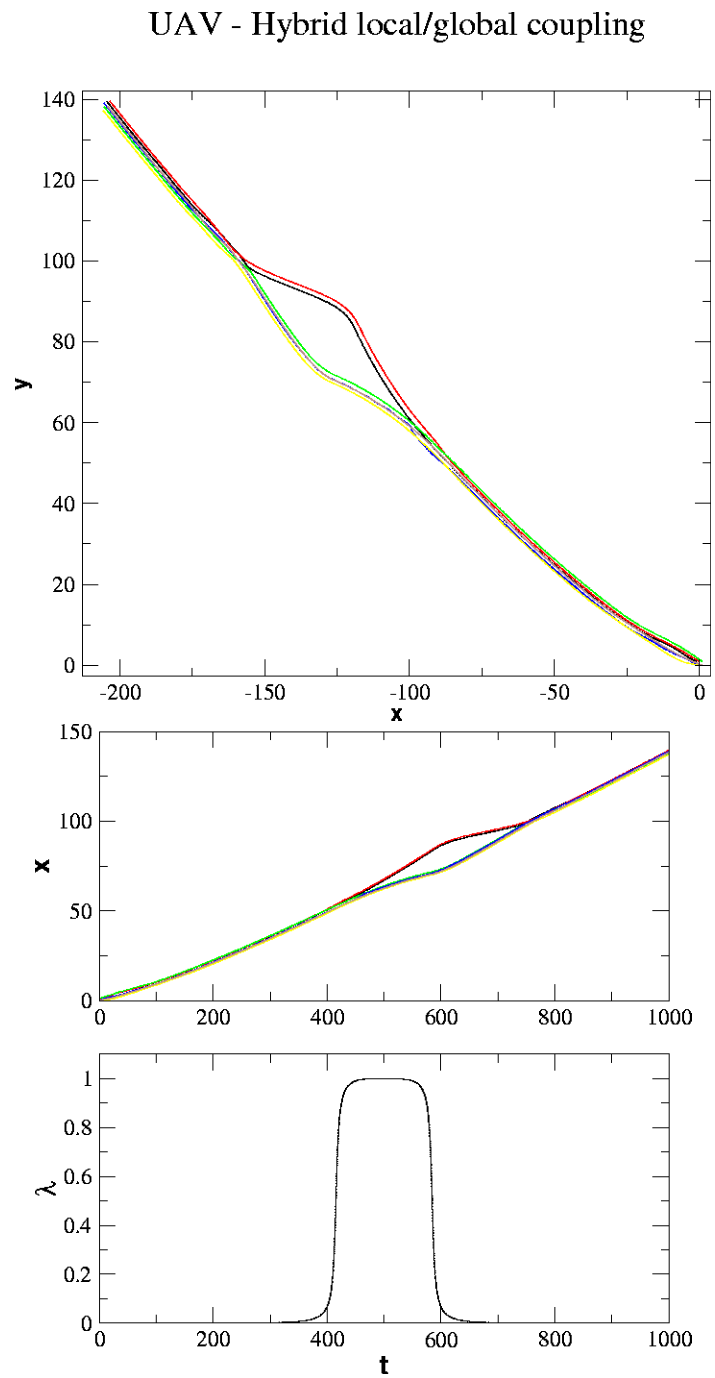

Figure 3. A simulation of the UAV model using the homotopy control law (12). The upper figure show a time-trace of the simulation in the $\mathbf{r} = (x, y)$ plane, the center plot shows x vs t, and the lower plot, λ vs t. Initially, $\lambda = 0$ and the global, group averaged control law is used. Agents start out in arbitrary directions, but soon organize into a unified, coherent swarm. Near $t = 400$, the homotopy parameter is turned on ($\lambda = 1$), which switches the control law to local coupling with leader following motion, and the swarm splits into two subswarms. Near $t = 600$, the homotopy parameter is switched off and global coupling is re-enabled, and a single swarm automatically reforms.

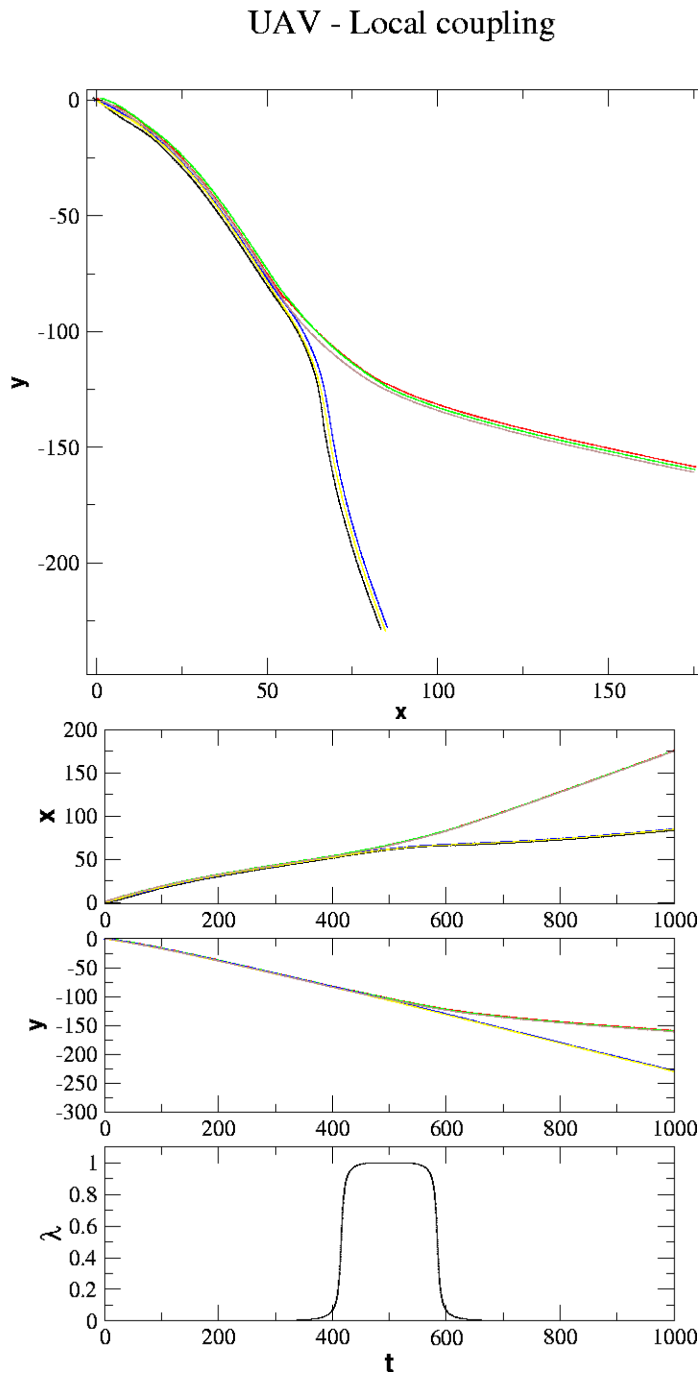

Figure 4. A simulation of the UAV model using local coupling only. The upper figure show a time-trace result of the simulation in the plane, while the lower plot shows r_x vs t, r_y vs t and λ vs t, respectively. Initially, the full swarm is in communication, $\lambda = 0$ and the locally coupled, group averaged control law is used. As in Fig. 3, agents soon organize into a unified, coherent swarm. Near $t = 400$, the homotopy parameter is turned on ($\lambda = 1$), which switches the control law to leader following motion, and the swarm splits into two subswarms. Near $t = 600$, the homotopy parameter is switched off and group averaged motion is re-enabled. Due to the fact that local coupling is used and the subswarm clusters are far apart, a single swarm does not reform in this case.

The character of the ideas in this section is similar in spirit to those presented in [23], where a force law is introduced with a global potential for target seeking, but where both gyroscopic and braking forces are used for collision avoidance.

5.1 *Target seeking control law.*

A target is considered here to be a fixed point in the plane that is specified ahead of time. In the context of UAVs, this implies that the position of a target would be preprogrammed before deployment, though an alternate possibility would be to communicate target coordinates to agents in flight. We introduce a target in the model by globally coupling the agents to an 'agent' that does not move. Let $\bar{r}$ be the fixed location of the target. The modified control law for target seeking is

$$u_k = \sum_{j \neq k} \left[c(|r_{jk}|, 0, w)(-\alpha \left(1 - \left(\frac{r_0}{|r_{jk}|}\right)^2\right)(r_{jk} \cdot y_k) \right] + \gamma\alpha \left(1 - \left(\frac{r_0}{|\bar{r}_k|}\right)^2\right)(\bar{r}_k \cdot y_k). \tag{15}$$

where $\bar{r}_k$ is the vector directed from the position of the k^{th} agent to $\bar{r}$, and γ is a weighting constant. Note that there is no term in this control law to align agents to a common heading. In fact, the only inter-agent term is the first term, involving the summation, which provides for collision avoidance. The cutoff function c implies that this term will have no effect if agents are outside of the cutoff radius w. The second term, which is global, steers individual agents toward the target. Though there are no terms to explicitly group agents, if the initial conditions are chosen so that the agents start in a group, then they will tend to stay together, as they collectively steer toward the target.

5.2 *Barrier avoidance*

The homotopy control law presented in section 4 can be used for barrier avoidance by splitting a swarm into two subswarms, which are steered independently around the barrier. The swarm splitting is achieved by explicitly changing the homotopy parameter λ. In contrast, we consider here an additional term for the control law, which applies an angular force to an agent when it is within sensing range of a barrier, where the position of the barrier is *a priori* unknown.

We restrict ourselves to the case of a convex and stationary barrier in $\mathbb{R}^2$. For our purposes, a barrier B is the convex hull defined by a set of m points $b_i \in \mathbb{R}^2$. The location of the barrier is not *a priori* known to an agent. Instead the barrier is detected whenever an agent is within range of any of the points defining the barrier, in which case we say that the agent is within range of the barrier.

Barrier avoidance logic is implemented in the model as follows. For each agent, at every time step of the simulation, we calculate a vector, the *average barrier direction vector*, and which may be the zero vector if the agent is not within a neighborhood of a barrier. The average barrier direction vector (directed from the k^{th} agent) is defined as,

$$v_k = \begin{cases} \frac{\sum_{i=1}^m [(b_i - y_k)c(|b_i - y_k|, 0, w)]}{\left|\sum_{i=1}^m [(b_i - y_k)c(|b_i - y_k|, 0, w)]\right|}, & \text{if } \left|\sum_{i=1}^m [(b_i - y_k)c(|b_i - y_k|, 0, w)]\right| \neq 0 \\ 0, & \text{otherwise} \end{cases}, \tag{16}$$

where $c(\cdot)$ is the cutoff function (8). The control law is then modified by adding the term $\pm(v_k \cdot y_k)s$, where the sign is chosen to be the sign of the expression $v_k \cdot y_k^{\perp}$. This term serves to steer the agent perpendicular to the direction of the average vector v_k, and the sign is chosen to steer the agent away from the the average direction of the barrier, relative to the current heading of the vehicle. This can result in a splitting of the swarm into two subswarms, with one subswarm going around one side of the barrier, and the other subswarm going around the other side.

5.3 Simulation results for target-seeking and barrier avoidance control law

We present the results of a simulation of the model using the control law 15 with the barrier avoidance control 16. Figure 5 shows the result of a typical simulation. A target point is located at $(x,y) = (400,0)$, and there is a hexagonal barrier centered at $(x,y) = (200,0)$, and is defined by the points $\{(200,2),(201,1),(201,-1),(200,-2),(199,-1),(199,1)\}$. As the swarm approaches the barrier, it again splits into two subswarms. All agents rejoin into a single swarm after the barrier is passed and continue on to the target. This time, upon arrival, the agents swarm in an irregular fashion about the target point. The irregular swarming about the target is due to the lack of inter-agent baseline controls implemented in this version of the control law.

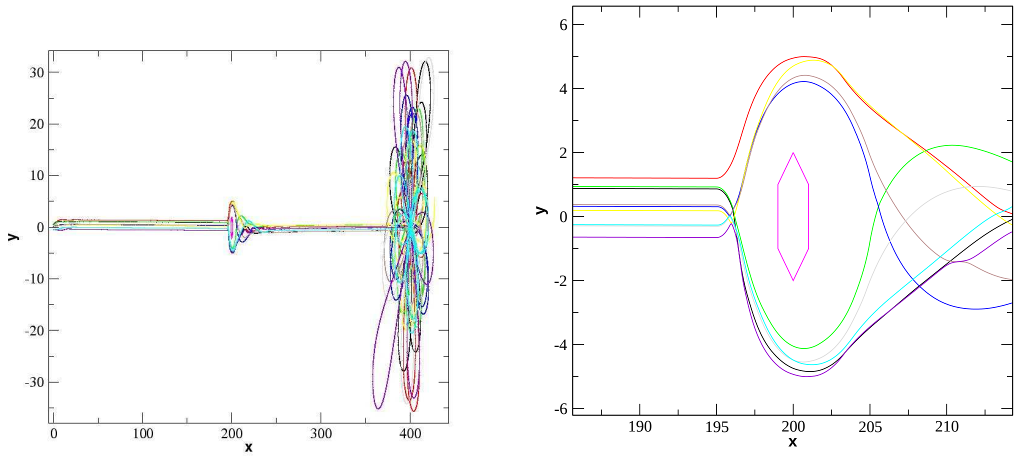

Figure 5. Results of a simulation using the control law of equation (15). The figure on the left shows a time-trace of the complete simulation. The swarm travels straight to the target point, which is at $(x,y) = (400,0)$, while avoiding the barrier centered at $(x,y) = (200,0)$. The figure on the right shows a detail of the barrier avoidance.

6 Discussion

The presented Frenet-Serret model and associated control laws exhibits robust and spontaneous coherent motion of a collection of n agents with controlled clustering for any smooth dynamical system. Such emergent behavior is important in obstacle and predator avoidance. Cluster formation from a coherent structure was done via a new type of control, which we introduced as a homotopy control. Using a simple central parameter, homotopy control provides an easily implementable method to create new emergent behavior from coherent structures. The model is robust in the sense that small perturbations of constituent agents of the swarm results in little or no change in the coherent motion of the swarm as a whole. We tested this by introducing additive noise into the simulations. At each time step, the positions and angles of the agents were perturbed with a small amount of noise. Noise was taken from a uniform distribution with mean zero. Results of simulations were qualitatively similar to those presented in section 4.

On the other hand, we also showed in section 5 that even with very loose coupling, involving only simultaneous target seeking, and where the only inter-agent coupling is via a collision avoidance term, ordered behavior can emerge, even when obstacle avoidance is taken into account.

Previous studies have focused on presenting unified coherent motion of a swarm. We have extended these results by presenting a method to automati-

cally transition to subswarm clusters, formed from an original larger swarm, and functioning independently. The spontaneous coherence implies that individual agents do not need to be manually controlled. Indeed, that is one main goal of such research; to find a set of (preferably simple) rules which will result in the desired behavior with a high degree of autonomy for the swarm, and with a minimum of external inputs.

There are some limitations to the currently considered model. When using only local coupling with the homotopy control law, there is no way to reunite subswarm clusters, and a separate mechanism would have to be introduced to do so. Additionally, the leader following model presented is asymmetric, in that there is a distinction between leader and follower agents. Thus, if a leader agent is disabled, the subswarm cluster is no longer controllable. A better approach would be to consider a symmetric control law that doesn't distinguish between leader and follower agents, but which maintains similar behaviors. This is the subject of ongoing research.

One obvious extension of the current model is to obtain a dictionary of useful controls which can be strung together in a similar fashion to what we have done with the homotopy control law, perhaps with multiple homotopy parameters, in order to obtain multiple emergent behaviors. Additionally, a stochastic control law, in which the swarm maintains a loose cohesiveness, while incorporating stochastic motion to avoid interception by predators, is also being explored as an extension.

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